

Millimetre and Radio Astronomy Techniques for Star-Formation Studies

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Radio and mm-bands

- Region of spectrum extends from 10MHz (30m wavelength) to 1THz(0.3mm) a range of 10^5
- Common technology across most of this range, Heterodyne techniques (convert electric fields to voltages, which can be amplified etc), parabolic 'dish' collectors
- Above detection method makes interferometry relatively easy.

Single Dish Instruments

Onsala 25m



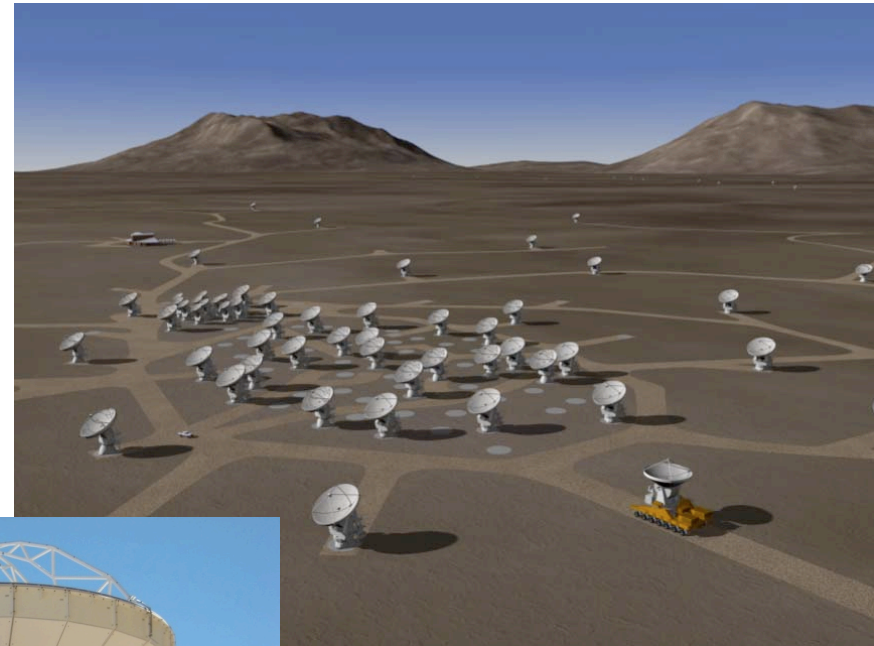
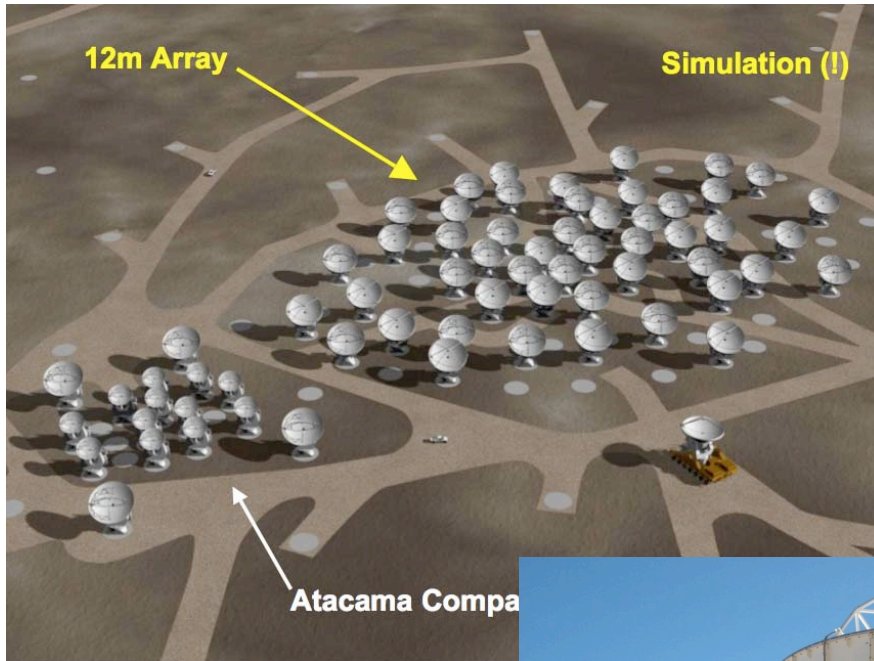
Onsala 20m

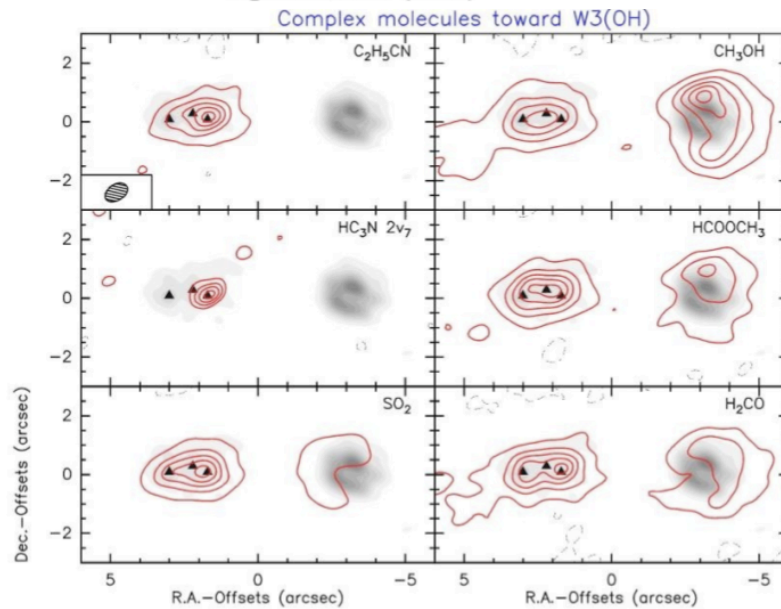
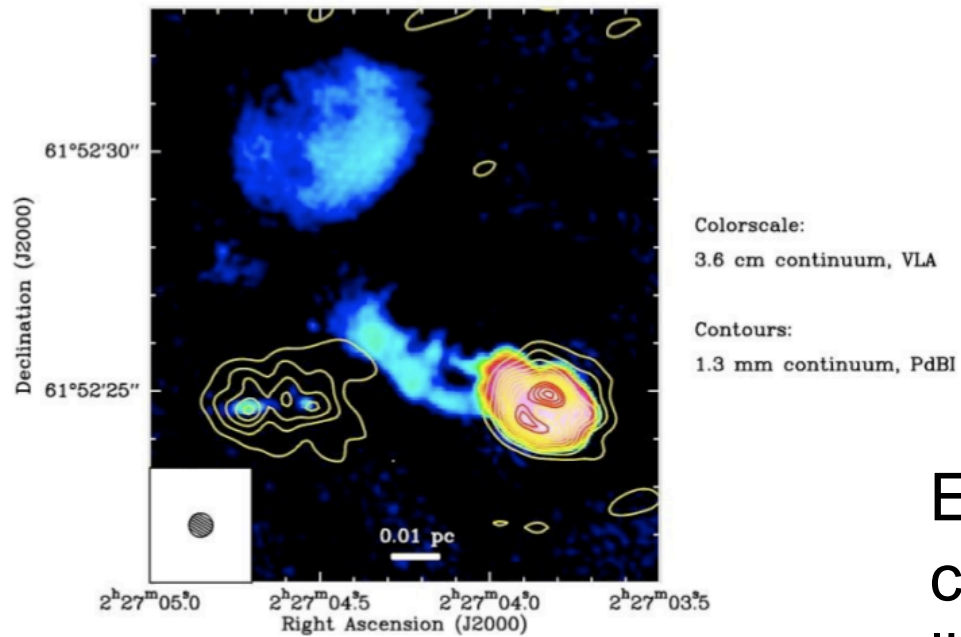


APEX telescope in
Chile at 5100m
elevation ALMA
site— joint MPIfR
Germany/Sweden/
ESO instrument



Interferometers-(e.g ALMA)





Examples of radio/mm
continuum and spectral
line observations of in
star formation region
W3OH –
Wyroski et al (1999)

SF studies in Radio/millimetre?

- Star formation regions often very obscured in optical and even IR, but usually transparent in radio/mm. Get full view of the matter in SF region, rather than just 'seeing the outer surface'.
- Can obtain very high resolution using interferometry- needed given small scales on which SF occurs
- Continuum and spectral line diagnostics of all major components of ISM ionised, atomic and molecular medium – also probe magnetic fields.
- Observed molecules are also the main source of cooling in dense molecular clouds.

Lecture Overview

- This lecture starts science techniques , next will continue and add observing methods. Today.....
- (1) First give overview of the brightness/intensity units we use in radio astronomy (not magnitudes but ‘Temperatures’ in K).
- (2) Then consider how we able to use observations to measure, fundamental properties such as gas/dust column and volume densities and temperatures – from continuum and line obs. Not complete but gives flavour

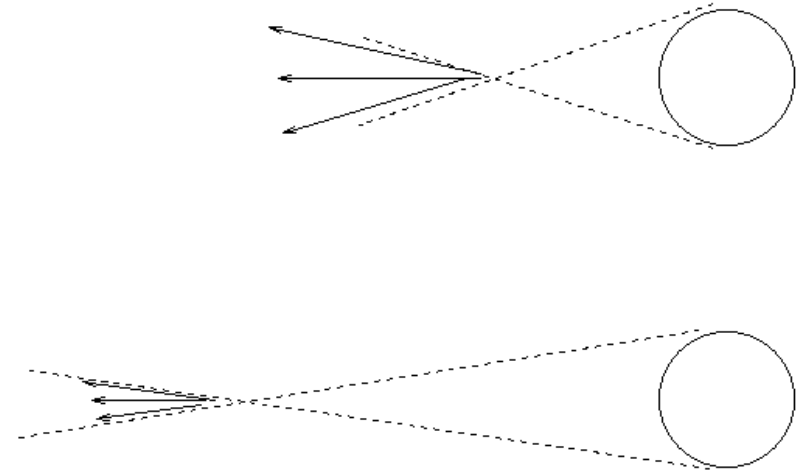
Closely follows material in ‘Introduction to Millimeter/Sub-millimeter astronomy’ by T. Wilson, March 2009, astro.ph 0903.0562, and textbook ‘Tools of Radio Astronomy’ Wilson, Rohlfs and Huttemeister. Also some viewgraphs borrowed from, 2007 IRAM summer school especially talk of P.Schilke (MPIfR Bonn)

PART 1 - Radio/mm temperature units and radiative transfer

Radiation propagation

I_ν = brightness or specific intensity, in $\text{W m}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$.

Stays constant along a ray in free-space. Photons have smaller flux in Wm^{-2} with increasing distance from object. However range of photon arrival directions which is narrower, hence flux per unit solid angle stays the same



Radio Astronomy Temperatures

$$B_{\nu}(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

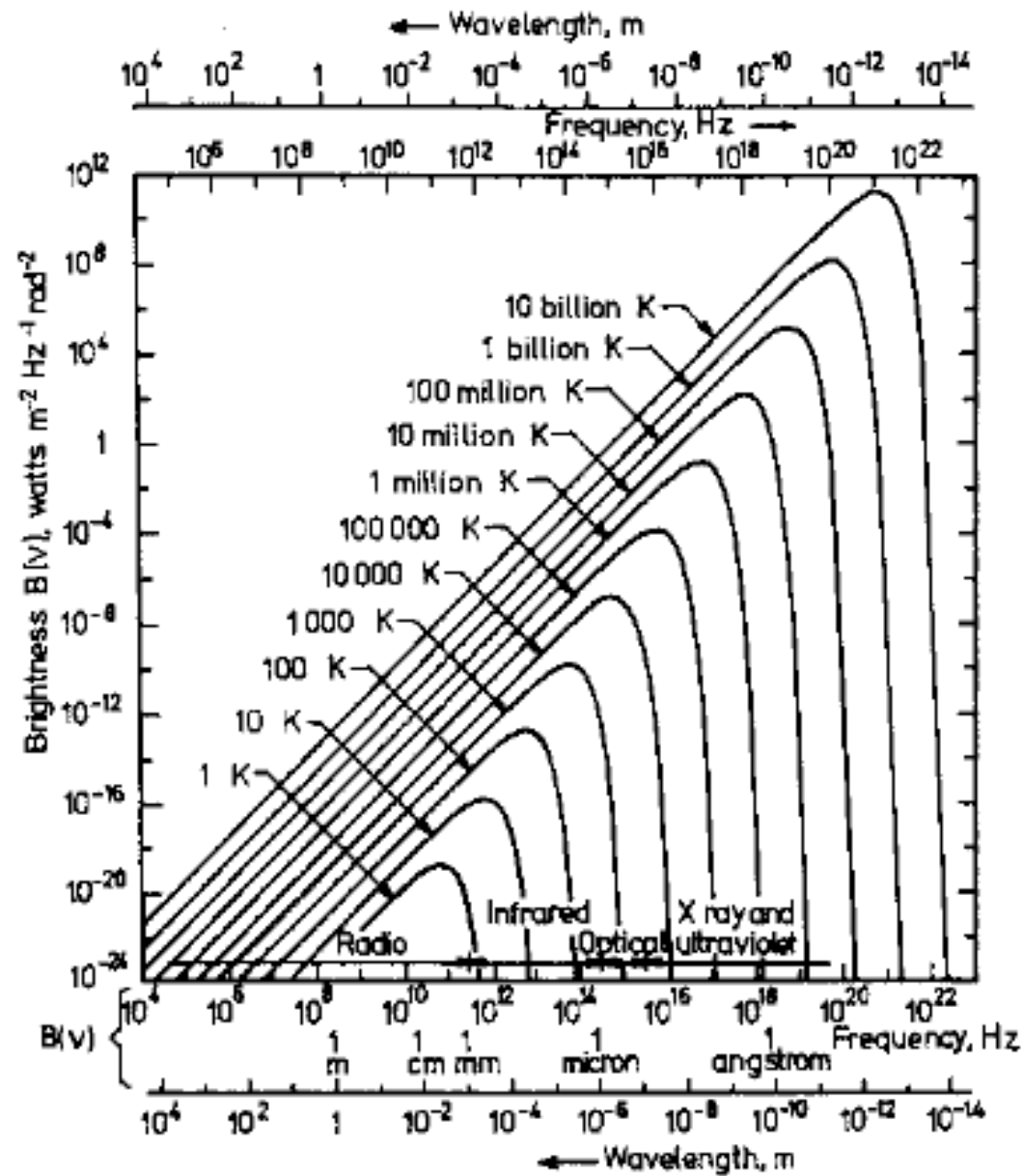
Plank radiation law for an emitting black body

$$B_{\text{RJ}}(\nu, T) = \frac{2\nu^2}{c^2} kT$$

Rayleigh-Jeans approximation to low frequency spectrum applies in radio/mm

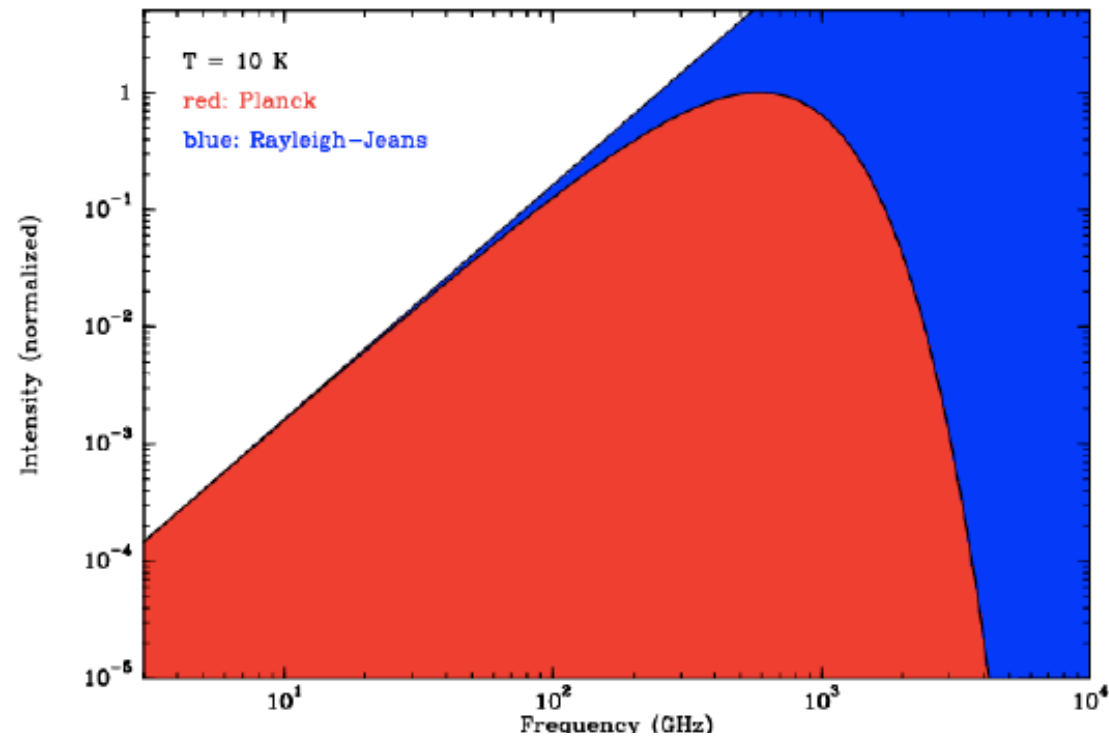
$$T_{\text{b}} = \frac{c^2}{2k} \frac{1}{\nu^2} I_{\nu} = \frac{\lambda^2}{2k} I_{\nu}$$

Given RG convenient to measure intensity of radiation received in units of 'brightness temperature' (K)



Planck black body spectra at different temperature

Rayleigh-Jeans Approximation



Applies well provided that

$$\frac{\nu}{\text{GHz}} \ll 20.84 \left(\frac{T}{\text{K}} \right)$$

Radio astronomers temperature obsession –Part 2

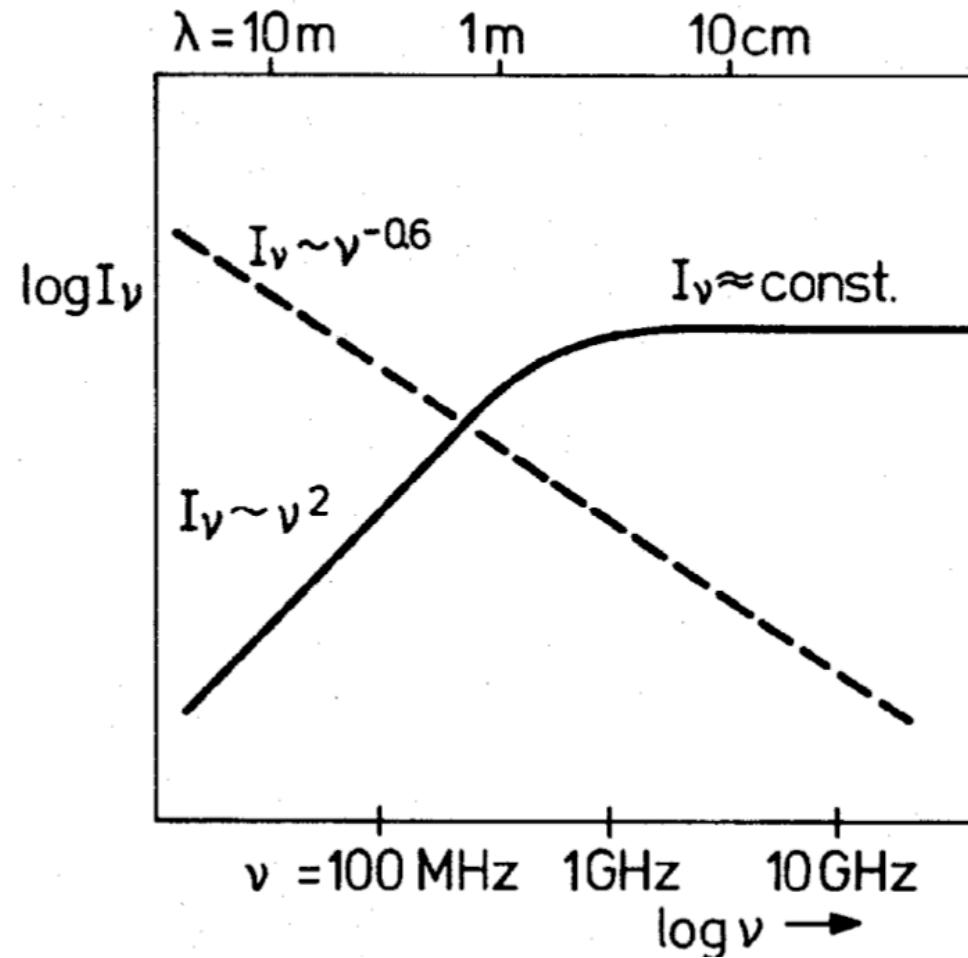
- Temperature units are also very practical to work in
- Compare power difference in milliWatts we get out of our receiver on blank sky (2.7K) with signal when we place a opaque hot load of known temperature (290K) in front of feed horn- so can directly calibrate astro signal mW to Kelvin.
- Because in radio astronomy we enormously amplify our signals and this amplification may be unstable, this is a more accurate method than trying to directly estimate Watts into receiver from radio source, from the Watts coming out.
- Finally noise added by receivers and atmosphere is related to their physical temperature.

PART 2 - Continuum and spectral line emission mechanisms – and how to get physical parameters

Radio-mm Continuum emission mechanisms

- Synchrotron, ultra-relativistic electrons in B fields, $I \propto \nu^{-\alpha}$, mostly at cm wavelengths (<20GHz), in extragalactic SF, from supernovae, measure star-death rate, but strongly correlated with star-birth, as shown via radio-FIR correlation (see next lecture) so can trace at high angular resolution star-formation
- Free-free emission, from ionised thermal gas (observable from 5-50GHz) i.e in HII regions powered by massive stars. Used to study massive stars in our galaxy and can also be used as a SF indicator in external galaxies.
- Dust emission at mm frequencies $\gg 100\text{GHz}$, has 'grey body' spectrum, get cloud temperatures, column densities, masses.

Synchrotron/Free-free emission



Radiation propagation

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + \varepsilon_\nu$$

$$\frac{\varepsilon_\nu}{\kappa_\nu} = B_\nu(T)$$

$$d\tau_\nu = -\kappa_\nu ds$$

$$-\frac{1}{\kappa_\nu} \frac{dI_\nu}{ds} = \frac{dI_\nu}{d\tau_\nu} = I_\nu - B_\nu(T)$$

$$I_\nu(s) = I_\nu(0) e^{-\tau_\nu(s)} + \int_0^{\tau_\nu(s)} B_\nu(T(\tau)) e^{-\tau} d\tau$$

For a uniform
medium

$$I_\nu(s) = I_\nu(0) e^{-\tau_\nu(s)} + B_\nu(T) (1 - e^{-\tau_\nu(s)})$$

If no background, received intensity versus frequency is Planck function multiplied by a frequency dependant opacity function.

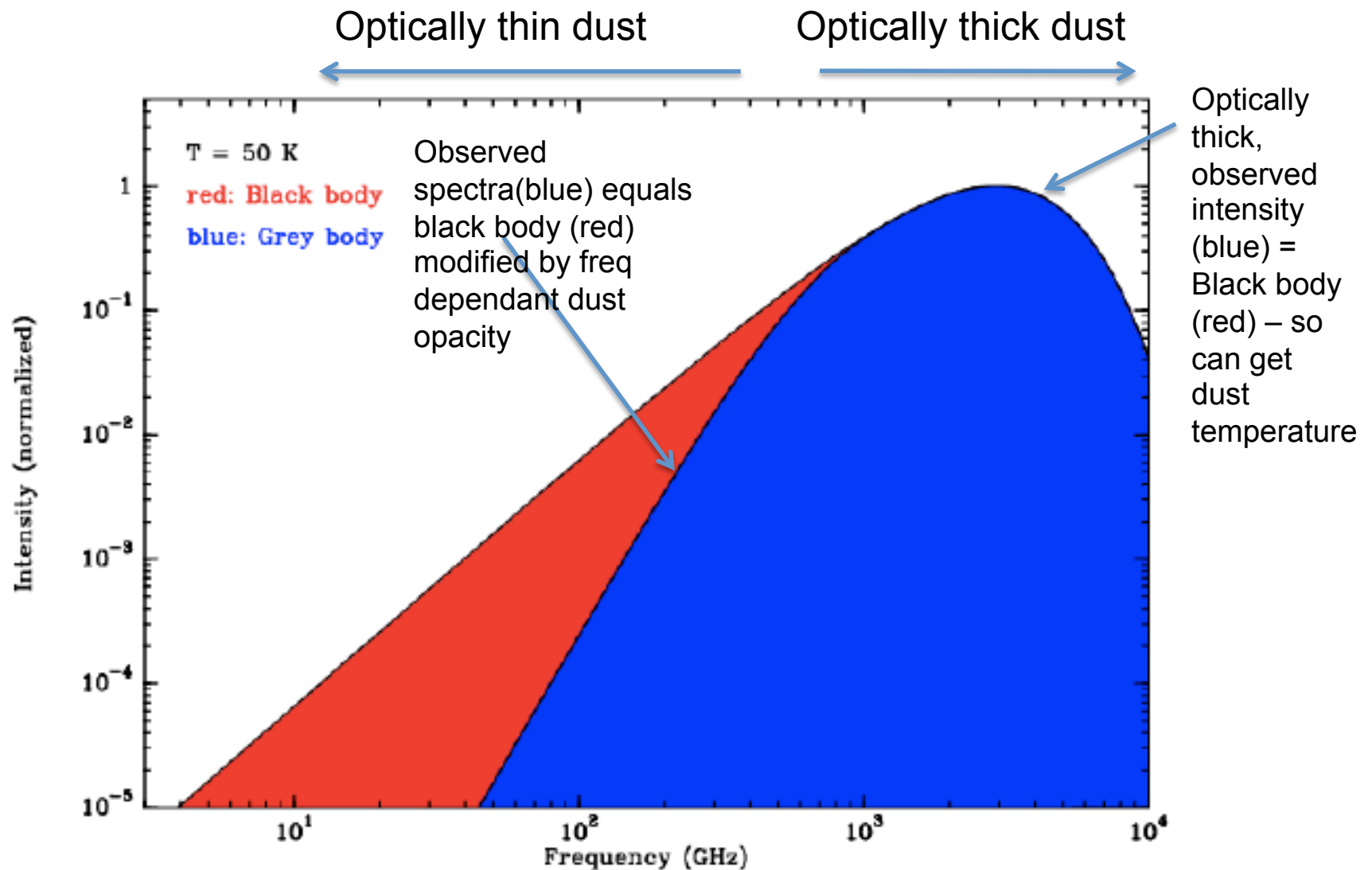
Radiation propagation

- Converting to temperature units

$$T_b(s) = T_b(0) e^{-\tau_\nu(s)} + \int_0^{\tau_\nu(s)} T(s) e^{-\tau} d\tau$$

For a uniform medium

$$T_b(s) = T_b(0) e^{-\tau_\nu(s)} + T (1 - e^{-\tau_\nu(s)})$$



Very steep $(\text{freq})^4$ dust spectra, so requires mm/sub-mm to detect dust continuum. At 100-200GHz see optically thin emission, at >500GHz (or IR), dust may be optically thick and we can directly measure dust temperature. Knowing this and comparing observed and black body curves at lower freq where dust is optically thin we can derive dust column density (and gas column using assumed gas/dust ratios).

Dust radiation

- Mass absorption coefficient

$$\kappa = \kappa_0 \left(\frac{\nu}{230 \text{ GHz}} \right)^\beta \text{ cm}^2 \text{ g}^{-1}$$

- κ_0 is typically $0.4 \text{ cm}^2 \text{ g}^{-1}$ in the ISM, but can vary with
 - Grain size
 - Grain properties (fluffy, ice mantle)
- β is around 2 in the ISM, and lower (1 or so) in disks (grain growth)

Dust radiation

- Dust opacity

$$\begin{aligned}\tau_{\text{dust}} &= \kappa_{\text{dust}} \frac{M_{\text{dust}}}{M_{\text{gas}}} M(\text{H}_2) N(\text{H}_2) \\ &= 3.3 \times 10^{-26} \kappa_{\text{dust}} N(\text{H}_2)\end{aligned}$$

- Grey body radiation as long as dust is optically thin

- Used for determination of

- Gas column densities
- Gas masses

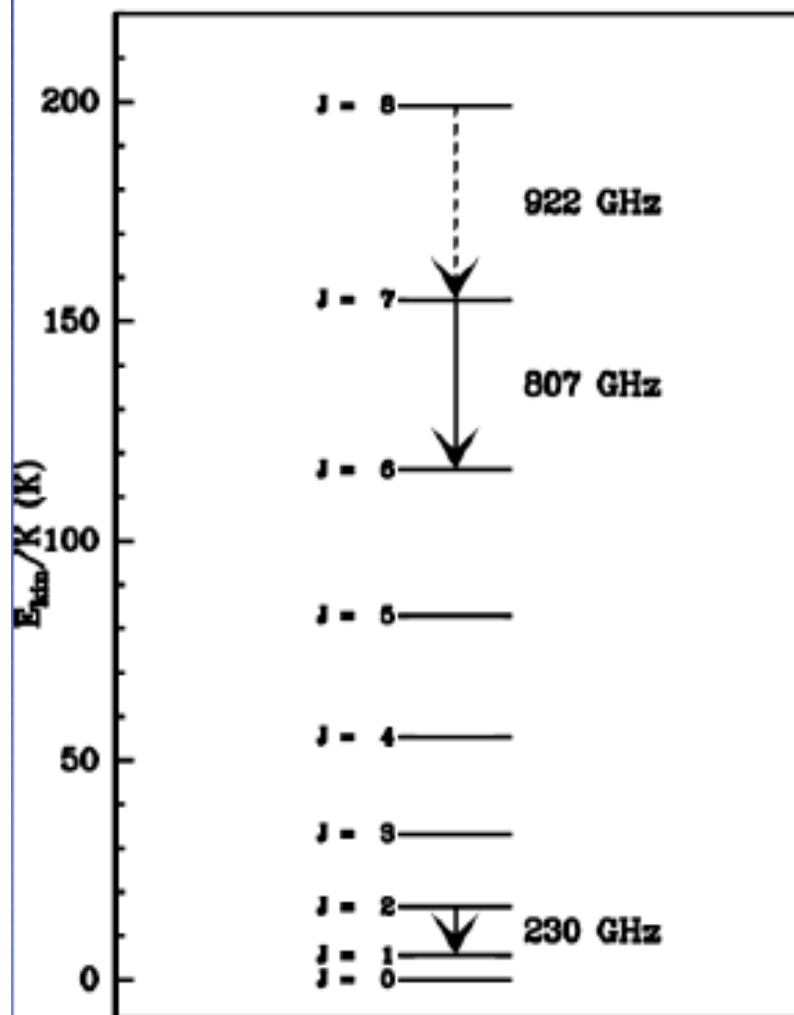
Intergrate $N(\text{H}_2)$ over face of clouds

Estimate T from optically thick obs at >500GHz or IR observations, then at few 100GHz compare observed intensity and that from black body, gives opacity at this freq, input κ_{dust} for that frequency (previous slide) then get $N(\text{H}_2)$.

Radio-mm Spectral Line emission

- Atomic Hydrogen (HI) emission at 1.4GHz (21cm)
- Recombination lines, transition between very high order n electronic transition in H and other (e.g C) atoms (cm and mm wavelengths)
- Atomic lines from electronic transitions rare but some from C⁺/CII (at 492 and 809GHz)
- Molecular lines, mostly from rotational transitions (few at cm wavelengths, e.g OH but most at mm wavelengths) Cover only this (slightly).

Simplest Linear Molecules



I,e CO, HCN, HCO⁺ etc, unlike H₂ have permanent electric dipole moment. Get lines at millimetre wavelengths from transitions between different quantum rotation states.

$$E = K J(J + 1)$$

Selection rule J change by -1
hence $\Delta E = 2K J$

CO(1-0) at 115 GHz,
CO(2-1) at 230 GHz
CO(3-2) at 345 GHz.....

Molecular probes of different gas density

- Definition of excitation temperature T_{ex} between levels 1 and 2

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp\left(\frac{-h\nu}{kT_{\text{ex}}}\right)$$

- Level populations determined by both coupling to radiation and thermal gas via collisions
- In general

$$T_{\text{ex}} = T_K \frac{T_b A_{21} + T_0 C_{21}}{T_K A_{21} + T_0 C_{21}}$$

Where T_K is gas kinetic temperature, T_b the ambient radiation temperature, $T_0 = h\nu/k$ line temperature, A_{21} the transition Einstein coefficient and C_{21} collision probabilities per particle

Molecular probes of different density

$$T_{\text{ex}} = T_K \frac{T_b A_{21} + T_0 C_{21}}{T_K A_{21} + T_0 C_{21}}$$

- C_{21} increases proportional to ambient density
At low density where radiation dominates collisions $C_{21} \ll A_{21}$, then $T_{\text{ex}} \approx T_b$ which can be $T_b = T_{\text{CMB}} = 2.7\text{K}$
- At high density $C_{21} \gg A_{21}$ then $T_{\text{ex}} \approx T_K$
- Density at which levels become thermalised is the *critical density* $n_{\text{crit}} \propto A_{21}^{-1}$

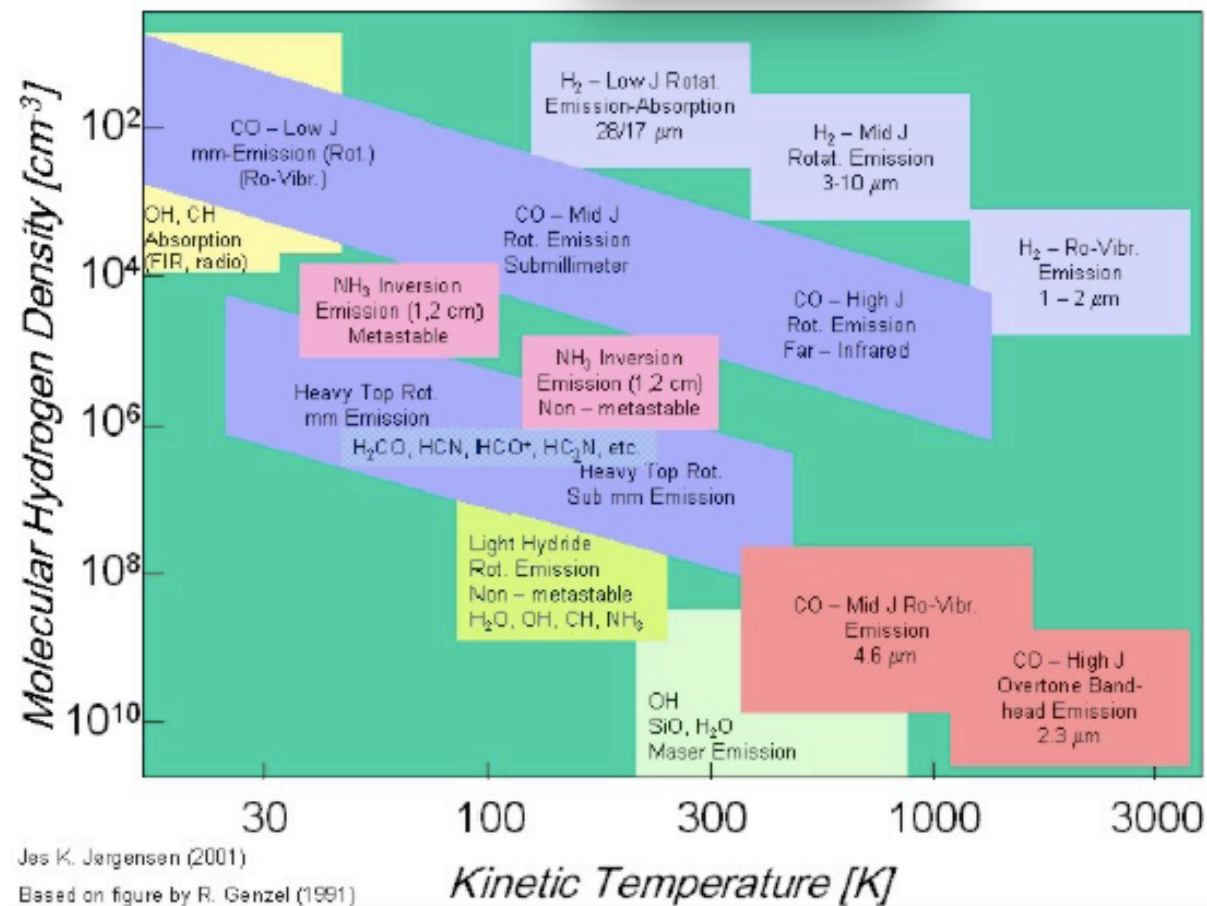
Molecular probes of different density

- If $n < n_{\text{crit}}$ then gas is sub-thermally excited, higher level not populated and line emission per molecule low.
- All gas with $n > n_{\text{crit}}$ radiates more efficiently with per molecule. As $n \gg n_{\text{crit}}$ this efficiency becomes independent of n .
- But for gas in a cloud complex with range of densities most of mass in gas is at lower densities Hence when observing a given molecule most emission comes from gas around a **typical density** of $n \approx n_{\text{crit}}$

Molecular probes of different T

- Given gas is thermally excited, each transition involves a upper level of a certain energy E_u , gas temperature must be high enough that this level is populated, i.e. $T > E_u/k$.
- But not too high or most molecules in higher states. Hence a given transition also probes gas at **typical temperature** $T \approx E_u/k$.

Molecular diagnostics



Choose molecular transition to pick out given T,n and then study kinematics of this gas

Typical medium
Temperature

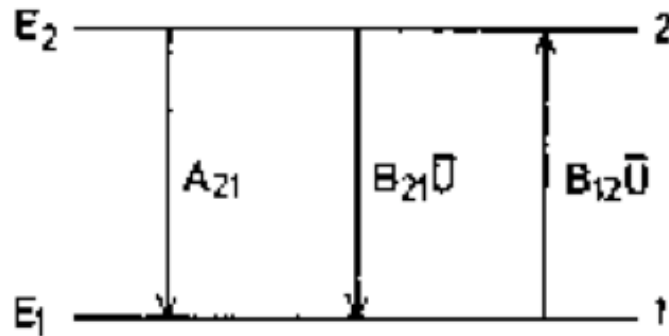
Critical
density
proportional
to A

Table 2. Parameters of the commonly observed short cm/mm molecular lines

Chemical ^a formula	Molecule name	Transition	ν/GHz	E_u/K^b	A_{ij}/s^{-1c}
H ₂ O	ortho-water*	$J_{K_a K_c} = 6_{16} - 5_{23}$	22.235253	640	1.9×10^{-9}
NH ₃	para-ammonia	$(J, K) = (1, 1) - (1, 1)$	23.694506	23	1.7×10^{-7}
NH ₃	para-ammonia	$(J, K) = (2, 2) - (2, 2)$	23.722634	64	2.2×10^{-7}
NH ₃	ortho-ammonia	$(J, K) = (3, 3) - (3, 3)$	23.870130	122	2.5×10^{-7}
SiO	silicon monoxide*	$J = 1 - 0, v = 2$	42.820587	3512	3.0×10^{-6}
SiO	silicon monoxide*	$J = 1 - 0, v = 1$	43.122080	1770	3.0×10^{-6}
SiO	silicon monoxide	$J = 1 - 0, v = 0$	43.423858	2.1	3.0×10^{-6}
CS	carbon monosulfide	$J = 1 - 0$	48.990964	2.4	1.8×10^{-6}
DCO ⁺	deuterated formylium	$J = 1 - 0$	72.039331	3.5	2.2×10^{-5}
SiO	silicon monoxide*	$J = 2 - 1, v = 2$	85.640456	3516	2.0×10^{-5}
SiO	silicon monoxide*	$J = 2 - 1, v = 1$	86.243442	1774	2.0×10^{-5}
H ¹³ CO ⁺	formylium	$J = 1 - 0$	86.754294	4.2	3.9×10^{-5}
SiO	silicon monoxide	$J = 2 - 1, v = 0$	86.846998	6.2	2.0×10^{-5}
HCN	hydrogen cyanide	$J = 1 - 0, F = 2 - 1$	88.631847	4.3	2.4×10^{-5}
HCO ⁺	formylium	$J = 1 - 0$	89.188518	4.3	4.2×10^{-5}
HNC	hydrogen isocyanide	$J = 1 - 0, F = 2 - 1$	90.663574	4.3	2.7×10^{-5}
N ₂ H ⁺	diazenylium	$J = 1 - 0, F_1 = 2 - 1,$ $F = 3 - 2$	93.173809	4.3	3.8×10^{-5}
CS	carbon monosulfide	$J = 2 - 1$	97.980968	7.1	2.2×10^{-5}
C ¹⁸ O	carbon monoxide	$J = 1 - 0$	109.782182	5.3	6.5×10^{-8}
¹³ CO	carbon monoxide	$J = 1 - 0$	110.201370	5.3	6.5×10^{-8}
CO	carbon monoxide	$J = 1 - 0$	115.271203	5.5	7.4×10^{-8}
H ₂ ¹³ CO	ortho-formaldehyde	$J_{K_a K_c} = 2_{12} - 1_{11}$	137.449959	22	5.3×10^{-5}
H ₂ CO	ortho-formaldehyde	$J_{K_a K_c} = 2_{12} - 1_{11}$	140.839518	22	5.3×10^{-5}
CS	carbon monosulfide	$J = 3 - 2$	146.969049	14.2	6.1×10^{-5}
C ¹⁸ O	carbon monoxide	$J = 2 - 1$	219.560319	15.9	6.2×10^{-7}
¹³ CO	carbon monoxide	$J = 2 - 1$	220.398714	15.9	6.2×10^{-7}
CO	carbon monoxide	$J = 2 - 1$	230.538001	16.6	7.1×10^{-7}
CS	carbon monosulfide	$J = 5 - 4$	244.935606	33.9	3.0×10^{-4}
HCN	hydrogen cyanide	$J = 3 - 2$	265.886432	25.5	8.5×10^{-4}
HCO ⁺	formylium	$J = 3 - 2$	267.557625	25.7	1.4×10^{-3}
HNC	hydrogen isocyanide	$J = 3 - 2$	271.981067	26.1	9.2×10^{-4}

Choose molecular species depending on n, T of gas you want to probe. Also must consider expected strength of line, depends on intrinsic strength (prop to A) + chemical abundance. Must be strong enough to detect, but may want to avoid being optically thick

Spectral Line Radiation transfer



$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp \left(- \frac{h\nu_0}{kT} \right)$$

DEFINITION of line
excitation temperature T ,
only in LTE does this equal
kinetic T

Recall from earlier the radiation transfer equation, which
equally applies to molecules just as continuum

$$I_\nu(s) = I_\nu(0) e^{-\tau_\nu(s)} + B_\nu(T) (1 - e^{-\tau_\nu(s)})$$

Where $B_\nu(T)$ is the Planck function that would be obtained
for a black body whose temperature equalled the line
excitation temperature

Measured Brightness Temp

$$T_{B\nu} = \frac{c^2}{2k\nu^2} (I_\nu(s) - I_\nu(0))$$

Remove spectral baseline so
line temp versus frequency is
value after removing
background continuum

$$T_{B\nu} = \frac{c^2}{2k\nu^2} [B_\nu(T) - B_{bg\nu}(T)] (1 - e^{-\tau_\nu(s)})$$

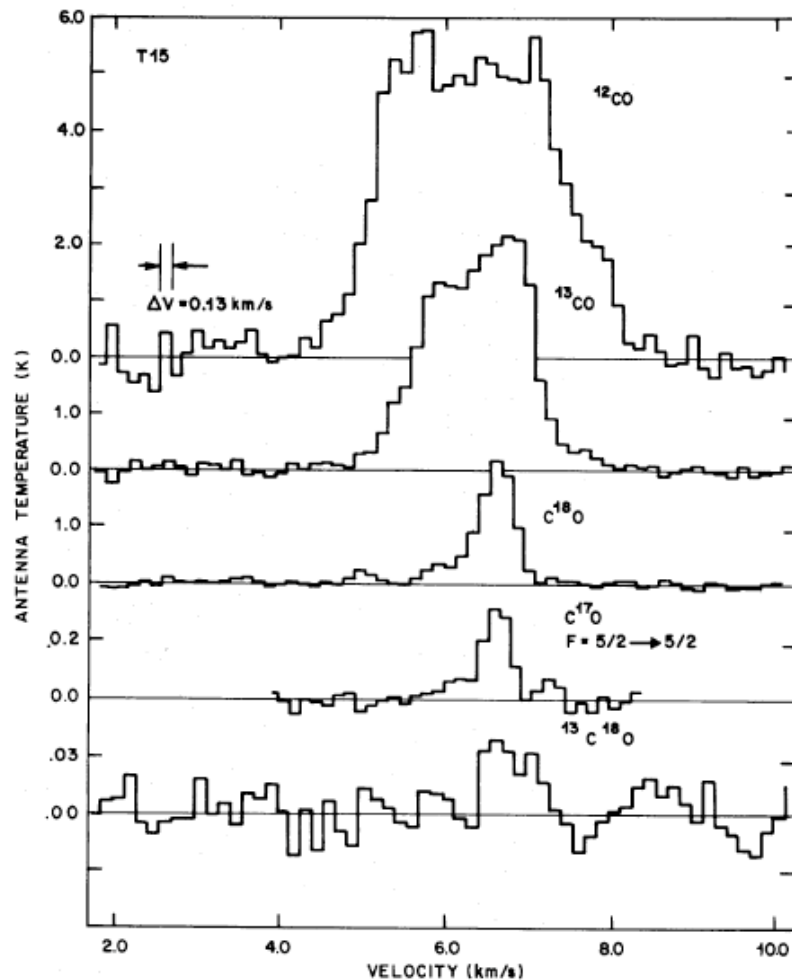
$$T_{B\nu} = T_o ([f(T) - f(T_{BG})] (1 - e^{-\tau_\nu(s)}))$$

The 'Detection'
equation

$$T_o = h\nu/k : \quad f(T) = (e^{(T_o/T)} - 1)^{-1}.$$

Funny looking $f(T)$ functions take into account limit
inaccuracies in the Rayleigh-Jeans approximation

Example CO – deriving gas temperature and opacity



Regular CO (12-C, 16-O) (1-0) transition. Often optically thick in ISM, can use brightness temperature at 'flat top' to estimate T using previous equations, here $T_B = 5.8\text{K}$, assuming $T_{BG} = 2.7\text{K}$, $T = 9.1\text{K}$

Then use an isomer e.g ^{13}CO which is optically thin assuming it has same excitation temp T as ^{12}CO .

Measuring its peak temperature and using detection equation gives an opacity here of $\tau = 1.2$

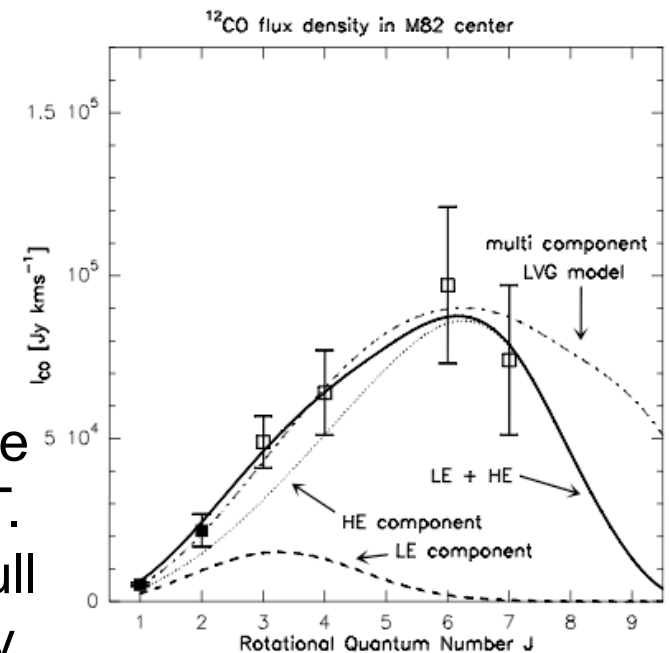
^{12}CO opacity can be estimated by expected $^{12}\text{CO}/^{13}\text{CO}$ ratio at this physical temperature, approx 25

More accurate T from fitting multiple transitions

- Assuming in LTE - good approx at low J for CO with its low critical total gas density (100cm^{-3}) and low excitation temp (5.6K for 1-0 transition).

$$N(J)/N(\text{total}) = \frac{(2J+1)}{Z} \exp \left[-\frac{h B_e J(J+1)}{kT} \right].$$

- N(J) first increases (because of level multiplicity term $(2J+1)$) then decreases because these high E levels not populated for this. Using optically thin line ratios say $^{13}\text{CO}(2-1)$ and $^{13}\text{CO}(1-0)$ estimate for T. Higher transitions (which depart from LTE) need full modelling to fit both T and n, i.e. use Large Velocity Gradient (LVG) methods



Column density

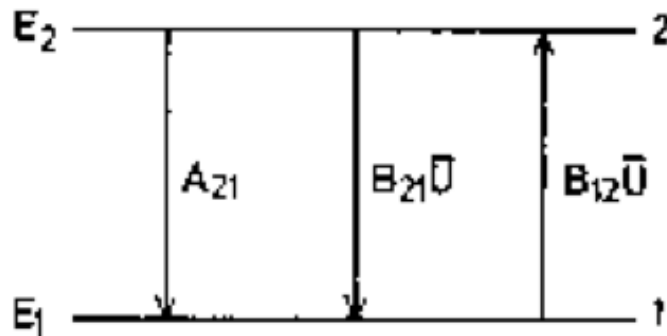
- To get column density observe optically thin line so that in detection equation

$$T_{B\nu} = T_o([f(T) - f(T_{BG})](1 - e^{-\tau_\nu(s)})$$

we can take $(1 - e^{-\tau}) \approx \tau$

- Then line brightness proportional to opacity through cloud. The $^{13}\text{CO}(1-0)$ is bright and is usually (marginally) optically thin so can be used.
- Opacity in this $^{13}\text{CO}(1-0)$ transition depends of total ^{13}CO col density and relative fraction of molecules in $J=0$ and $J=1$ levels which in turn depends on T .

Absorption coefficient



$$\frac{dI_\nu}{ds} = -\frac{h\nu_0}{c} (N_1 B_{12} - N_2 B_{21}) I_\nu \varphi(\nu) + \frac{h\nu_0}{4\pi} N_2 A_{21} \varphi(\nu)$$

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + \varepsilon_\nu$$

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp\left(-\frac{h\nu_0}{kT}\right)$$

DEFINITION of
line excitation
temperature T , in
LTE only equals
kinetic T

$$\kappa_\nu = \frac{c^2}{8\pi} \frac{1}{\nu_0^2} \frac{g_2}{g_1} N_1 A_{21} \left[1 - \exp\left(-\frac{h\nu_0}{kT}\right) \right] \varphi(\nu)$$

Column density

$$\kappa_\nu = \frac{c^2}{8\pi} \frac{1}{\nu_0^2} \frac{g_2}{g_1} N_1 A_{21} \left[1 - \exp\left(-\frac{h\nu_0}{kT}\right) \right] \varphi(\nu)$$

Here N_1 refers to number density in lower transition

Integrate κ along line of sight to get opacity, integrate in freq/velocity over line profile, substitute value of $h\nu/k$ for $^{13}\text{CO}(1-0)$ and rearrange

$$N_l = 93.5 \frac{g_l \nu^3}{g_u A_{ul}} \frac{1}{[1 - \exp(-4.80 \times 10^{-2} \nu / T_{\text{ex}})]} \int \tau dv$$

Here (sorry!) N_l now refers to column density in lower state

This is column density for one rotation state of molecule. To get *total* column density for molecule, summing over all states requires knowing 'partition function' (which is a number versus T , for CO is ratio of total density in a molecule to density of $J=0$ state).

$$Z = \sum_{J=0}^{\infty} (2J+1) \exp\left[-\frac{h B_e J(J+1)}{kT}\right]$$

Where B_e is a constant for molecule

^{13}CO Column density

- For lowest state $l=j=0$, then $N_{\text{tot}} = Z N_l$, can be approximated

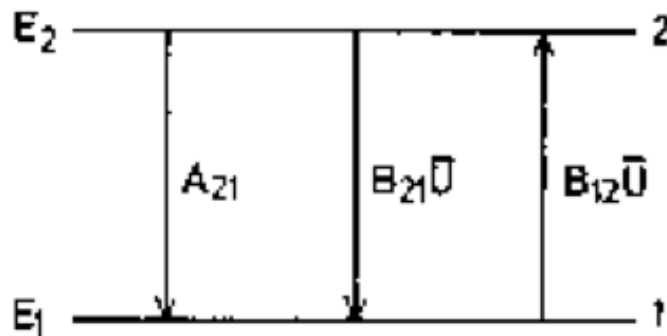
$$Z \approx \frac{kT}{h B_e} \quad \text{for } h B_e \ll kT .$$

$$N(\text{total})_{\text{CO}}^{13} = 3.0 \times 10^{14} \frac{T \int \tau^{13}(v) dv}{1 - \exp \{-5.3/T\}}$$

Formula for
 $^{13}\text{CO}(1-0)$
observations

- For optically thin lines, the integral is just velocity integrated line emission (units K km s^{-1}), but the final estimate still depends on T (and is proportional to T for $T \gg 5.3\text{K}$)
need some estimate of T from earlier methods.

Spectral Line Basics



$$g_1 B_{12} = g_2 B_{21}$$

$$A_{21} = \frac{8\pi h \nu_0^3}{c^3} B_{21}$$

$$\frac{dI_\nu}{ds} = -\frac{h\nu_0}{c} (N_1 B_{12} - N_2 B_{21}) I_\nu \varphi(\nu) + \frac{h\nu_0}{4\pi} N_2 A_{21} \varphi(\nu)$$

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + \varepsilon_\nu$$

$$\kappa_\nu = \frac{h\nu_0}{c} N_1 B_{12} \left(1 - \frac{g_1 N_2}{g_2 N_1} \right) \varphi(\nu)$$

$$\varepsilon_\nu = \frac{h\nu_0}{4\pi} N_2 A_{21} \varphi(\nu)$$

$$\frac{\epsilon_\nu}{\kappa_\nu} = \frac{2h\nu^3}{c^2} \left(\frac{g_2 N_1}{g_1 N_2} - 1 \right)^{-1} \quad \boxed{\frac{\epsilon_\nu}{\kappa_\nu} = B_\nu(T)}$$

$$\boxed{B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}}$$

$$\boxed{\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp \left(-\frac{h\nu_0}{kT} \right)}$$

DEFINITION of line
excitation temperature T, in
LTE only equals kinetic T

$$\boxed{\kappa_\nu = \frac{c^2}{8\pi} \frac{1}{\nu_0^2} \frac{g_2}{g_1} N_1 A_{21} \left[1 - \exp \left(-\frac{h\nu_0}{kT} \right) \right] \varphi(\nu)}$$

Conversion to total gas column density

- Calibrate using UV and optical obs and get

$$N_H = 1.0 \times 10^6 N_{CO}^{13} \text{ cm}^{-2}$$

where $N_H = N_{HI} + 2N_{H2}$. Usually consider totally molecular so $N_{H2} = 5 \times 10^5 N_{CO}^{13} \text{ cm}^{-2}$

- See Stahler & Palla for derivation. Use UV obs of stars behind diffuse clouds to get relation of N_H to $E(B-V)$ hence A_V . In denser clouds can measure N_{CO}^{13} and estimate A_V from star counts, hence finally relate N_H to N_{CO}^{13}
- ^{13}CO observations good tracer of *total* H_2 molecular column density. Use column densities in other molecules HCN, HCO+ to trace gas at particular n, T and chemical state. Analysis more difficult, since further from LTE, needs i.e LVG modelling – but standard software exists on web

Next lectures

- Next time more detail on how a radio telescope works and how continuum and line temperatures on the sky measured for single dish obs.
- On Friday expand to discuss interferometry leading onto ALMA.